5.387 sum_of_increments

DESCRIPTION LINKS

Origin [86]

Constraint sum_of_increments(VARIABLES, LIMIT)

Synonyms increments_sum, incr_sum, sum_incr, sum_increments.

Arguments

VARIABLES	:	collection(var−dvar)
LIMIT	:	dvar

Restrictions

required(VARIABLES, var)
VARIABLES.var ≥ 0
LIMIT ≥ 0

Purpose Given a collection of variables VARIABLES which can only be assigned non negative values, and a variable LIMIT, enforce the condition VARIABLES[1].var + $\sum_{i=2}^{|\text{VARIABLES}|} \max(\text{VARIABLES}[i].\text{var} - \text{VARIABLES}[i-1].\text{var}, 0) \leq \text{LIMIT}$. VARIABLES[1].var stands from the fact that we assume an additional implicit 0 before the first variable (i.e., VARIABLES[1].var $= \max(\text{VARIABLES}[1].\text{var} - 0, 0)$).

Example $(\langle 4, 4, 3, 4, 6\rangle, 7)$

The sum_of_increments constraint holds since we have that $4 + \max(4-4,0) + \max(3-4,0) + \max(4-3,0) + \max(6-4,0) \leq 7$.

Typical

|VARIABLES| > 2
range(VARIABLES.var) > 1
maxval(VARIABLES.var) > 0
LIMIT > 0
LIMIT $\leq$ |VARIABLES|$*$range(VARIABLES.var)/2

Symmetries

- One and the same constant can be added to VARIABLES.var and to LIMIT.
- Items of VARIABLES can be reversed.
- LIMIT can be increased.

Arg. properties

- Prefix-contractible wrt. VARIABLES.
- Suffix-contractible wrt. VARIABLES.

Usage The sum_of_increments was initially motivated by the problem of decomposing a matrix of non-negative integers into a positive linear combination of matrices consisting of only zeros and ones, where the ones occur consecutively in each row.

Algorithm

A $O(|\texttt{VARIABLES}|)$ bound-consistency filtering algorithm for the `sum_of_increments` constraint is described in [86].

Reformulation

The following reformulations are provided in [86]. Assuming $\texttt{VARIABLES}[0].\texttt{var}$ is defined as 0 (i.e., a zero is added before the first variable of the `VARIABLES` collection) we have:

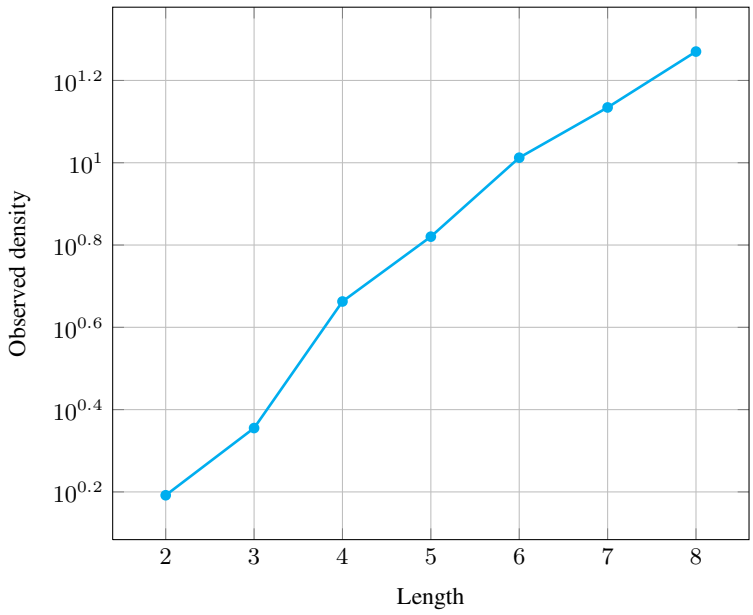
- $\sum_{i=1}^{|\texttt{VARIABLES}|} S_i \leq \texttt{LIMIT}$ with $D_i = \texttt{VARIABLES}[i].\texttt{var} - \texttt{VARIABLES}[i-1].\texttt{var}$ and $S_i = \max(D_i, 0)$ $(1 \leq i \leq |\texttt{VARIABLES}|)$.
- $\sum_{i=1}^{|\texttt{VARIABLES}|} S_i \leq \texttt{LIMIT}$ with $\texttt{VARIABLES}[i].\texttt{var} - \texttt{VARIABLES}[i-1].\texttt{var} \leq S_i$ and $S_i \in [0, \overline{\texttt{LIMIT}}]$ $(1 \leq i \leq |\texttt{VARIABLES}|)$.

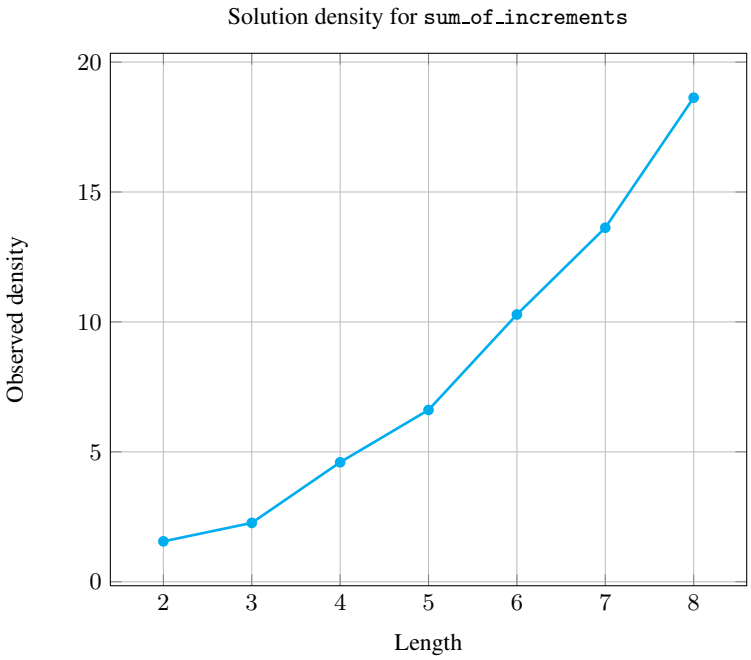
Counting

Length (n)	2	3	4	5	6	7	8
Solutions	14	145	2875	51415	1210104	28573741	801944469

Number of solutions for `sum_of_increments`: domains $0..n$

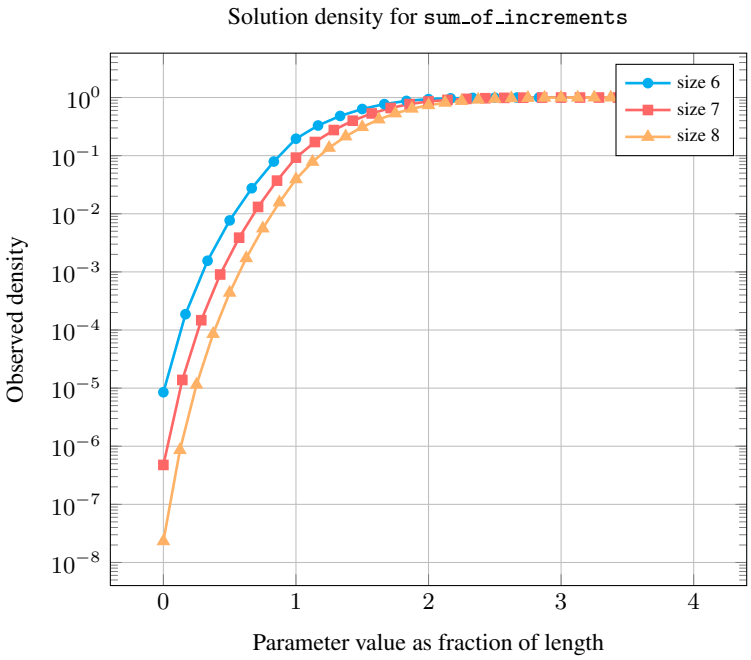
Solution density for `sum_of_increments`

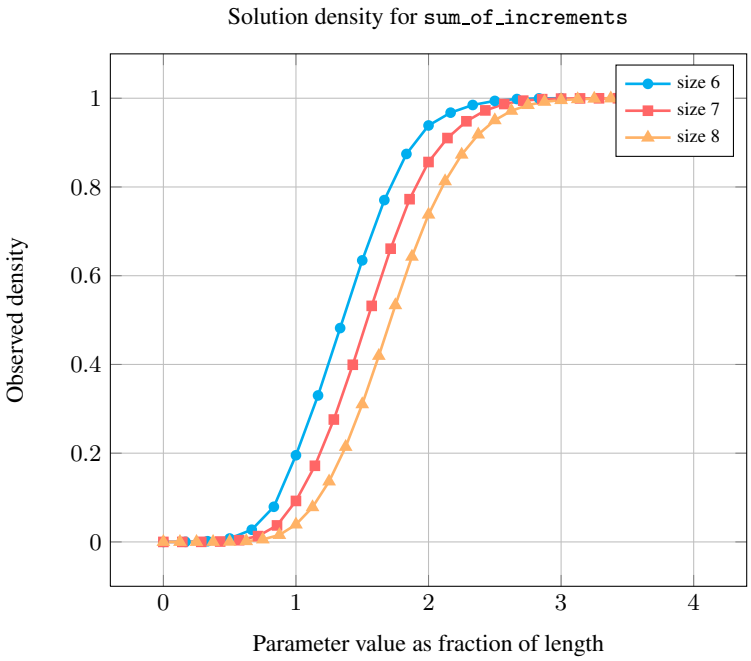



Solution density for sum_of_increments
Observed density
Length
0
5
10
15
20
2
3
4
5
6
7
8

Length (n)		2	3	4	5	6	7	8
Total		14	145	2875	51415	1210104	28573741	801944469
Parameter value	0	1	1	1	1	1	1	1
	1	4	7	11	16	22	29	37
	2	9	23	51	101	183	309	493
	3	-	54	156	396	904	1891	3679
	4	-	60	375	1167	3235	8135	18835
	5	-	-	485	2848	9318	27483	74143
	6	-	-	563	4263	22981	77947	240751
	7	-	-	608	5568	38836	193742	675244
	8	-	-	625	6616	56703	359880	1688427
	9	-	-	-	7314	74658	578511	3369015
	10	-	-	-	7650	90639	837441	5865915
	11	-	-	-	7720	102875	1115687	9220695
	12	-	-	-	7755	110425	1386029	13354545
	13	-	-	-	-	113827	1619993	18051195
	14	-	-	-	-	115857	1795694	22965651
	15	-	-	-	-	116942	1908968	27670800
	16	-	-	-	-	117437	1988222	31755573
	17	-	-	-	-	117612	2039616	34989993
	18	-	-	-	-	117649	2069933	37574073
	19	-	-	-	-	-	2085763	39526569
	20	-	-	-	-	-	2092817	40912205
	21	-	-	-	-	-	2095436	41827847
	22	-	-	-	-	-	2096360	42386387
	23	-	-	-	-	-	2096822	42700112
	24	-	-	-	-	-	2097032	42865683
	25	-	-	-	-	-	-	42953199
	26	-	-	-	-	-	-	43002171
	27	-	-	-	-	-	-	43027581
	28	-	-	-	-	-	-	43039551
	29	-	-	-	-	-	-	43044507
	30	-	-	-	-	-	-	43046215
	31	-	-	-	-	-	-	43046656
	32	-	-	-	-	-	-	43046721

Solution count for `sum_of_increments`: domains $0..n$





Keywords **characteristic of a constraint:** difference, sum.
constraint type: predefined constraint.
filtering: bound-consistency.