5.396 symmetric_alldifferent_except_0

DESCRIPTION LINKS

Origin Derived from symmetric_alldifferent

Constraint symmetric_alldifferent_except_0(NODES)

Synonyms symmetric_alldiff_except_0, symmetric_alldistinct_except_0, symm_alldifferent_except_0, symm_alldiff_except_0, symm_alldistinct_except_0.

Argument NODES : collection(index−int, succ−dvar)

Restrictions
required(NODES, [index, succ])
NODES.index ≥ 1
NODES.index $\leq$ |NODES|
distinct(NODES, index)
NODES.succ ≥ 0
NODES.succ $\leq$ |NODES|

Purpose

Enforce the following three conditions:

1. $\forall i \in [1, |\text{NODES}|], \forall j \in [1, |\text{NODES}|], (j \neq i)$: $\text{NODES}[i].\text{succ} = 0 \vee \text{NODES}[j].\text{succ} = 0 \vee \text{NODES}[i].\text{succ} \neq \text{NODES}[j].\text{succ}$.
2. $\forall i \in [1, |\text{NODES}|] : \text{NODES}[i].\text{succ} \neq i$.
3. $\text{NODES}[i].\text{succ} = j \wedge j \neq i \wedge j \neq 0 \Leftrightarrow \text{NODES}[j].\text{succ} = i \wedge i \neq j \wedge i \neq 0$.

Example

$$\left(\left\langle\begin{array}{ll} \text{index}-1 & \text{succ}-3, \\ \text{index}-2 & \text{succ}-0, \\ \text{index}-3 & \text{succ}-1, \\ \text{index}-4 & \text{succ}-0 \end{array}\right\rangle\right)$$

The symmetric_alldifferent_except_0 constraint holds since:

- $\text{NODES}[1].\text{succ} = 3 \Leftrightarrow \text{NODES}[3].\text{succ} = 1$,
- $\text{NODES}[2].\text{succ} = 0$ and value 2 is not assigned to any variable.
- $\text{NODES}[4].\text{succ} = 0$ and value 4 is not assigned to any variable.

Given 3 successor variables that have to be assigned a value in interval $[0, 3]$, the solutions to the symmetric_alldifferent_except_0 ($\langle \text{index}-1\ \text{succ}-s_1, \text{index}-2\ \text{succ}-s_2, \text{index}-3\ \text{succ}-s_3\rangle$) constraint are $\langle 1\ 0, 2\ 0, 3\ 0\rangle$, $\langle 1\ 0, 2\ 3, 3\ 2\rangle$, $\langle 1\ 2, 2\ 1, 3\ 0\rangle$, and $\langle 1\ 3, 2\ 0, 3\ 1\rangle$.

Given 4 successor variables that have to be assigned a value in interval $[0, 3]$, the solutions to the symmetric_alldifferent_except_0 ($\langle \text{index}-1\ \text{succ}-s_1, \text{index}-2\ \text{succ}-s_2, \text{index}-3\ \text{succ}-s_3, \text{index}-4\ \text{succ}-s_4\rangle$) constraint are $\langle 1\ 0, 2\ 0, 3\ 0, 4\ 0\rangle$, $\langle 1\ 0, 2\ 0, 3\ 4, 4\ 3\rangle$, $\langle 1\ 0, 2\ 3, 3\ 2, 4\ 0\rangle$, $\langle 1\ 0, 2\ 4, 3\ 0, 4\ 2\rangle$, $\langle 1\ 2, 2\ 1, 3\ 0, 4\ 0\rangle$, $\langle 1\ 2, 2\ 1, 3\ 4, 4\ 3\rangle$, $\langle 1\ 3, 2\ 0, 3\ 1, 4\ 0\rangle$, $\langle 1\ 3, 2\ 4, 3\ 1, 4\ 2\rangle$, $\langle 1\ 4, 2\ 0, 3\ 0, 4\ 1\rangle$, $\langle 1\ 4, 2\ 3, 3\ 2, 4\ 1\rangle$.

All solutions Figure 5.752 gives all solutions to the following non ground instance of the `symmetric_alldifferent_except_0` constraint: $S_1 \in [0..5]$, $S_2 \in [1..3]$, $S_3 \in [1..4]$, $S_4 \in [0..3]$, $S_5 \in [0..2]$, `symmetric_alldifferent_except_0`$(\langle 1\ S_1, 2\ S_2, 3\ S_3, 4\ S_4, 5\ S_5\rangle)$.

① $(\langle 0_1, 3_2, 2_3, 0_4, 0_5\rangle)$
② $(\langle 2_1, 1_2, 4_3, 3_4, 0_5\rangle)$
③ $(\langle 4_1, 3_2, 2_3, 1_4, 0_5\rangle)$
④ $(\langle 5_1, 3_2, 2_3, 0_4, 1_5\rangle)$

Figure 5.752: All solutions corresponding to the non ground example of the `symmetric_alldifferent_except_0` constraint of the **All solutions** slot (the `index` attribute is displayed as indices of the `succ` attribute)

Typical

$|$`NODES`$| \geq 4$
`minval`(`NODES.succ`) $= 0$
`maxval`(`NODES.succ`) > 0

Symmetry Items of `NODES` are permutable.

Usage Within the context of sport scheduling, `NODES`$[i]$`.succ` $= j$ $(i \neq 0, j \neq 0, i \neq j)$ is interpreted as the fact that team i plays against team j, while `NODES`$[i]$`.succ` $= 0$ $(i \neq 0)$ is interpreted as the fact that team i does not play at all.

Algorithm An arc-consistency filtering algorithm for the `symmetric_alldifferent_except_0` constraint is described in [131, 130]. The algorithm is based on the following facts:

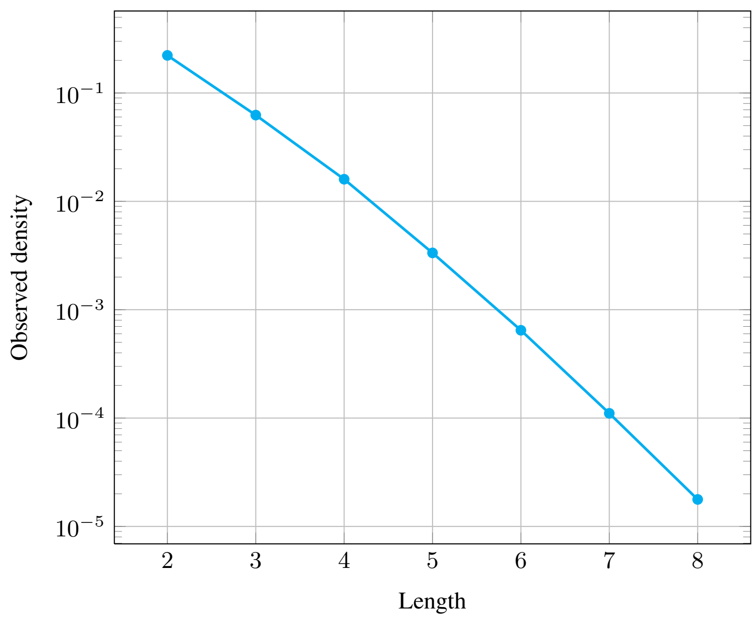
- First, one can map solutions to the `symmetric_alldifferent_except_0` constraint to perfect (g, f)-matchings in a non-bipartite graph derived from the domain of the variables of the constraint where $g(x) = 0$, $f(x) = 1$ for vertices x which have 0 in their domain, and $g(x) = f(x) = 1$ for all the remaining vertices. A *perfect* (g, f)-*matching* $\mathcal{M}$ of a graph is a subset of edges such that every vertex x is incident with the number of edges in $\mathcal{M}$ between $g(x)$ and $f(x)$.
- Second, Gallai-Edmonds decomposition [179, 150] allows to find out all edges that do not belong to any perfect (g, f)-matchings, and therefore prune the corresponding variables.

Counting

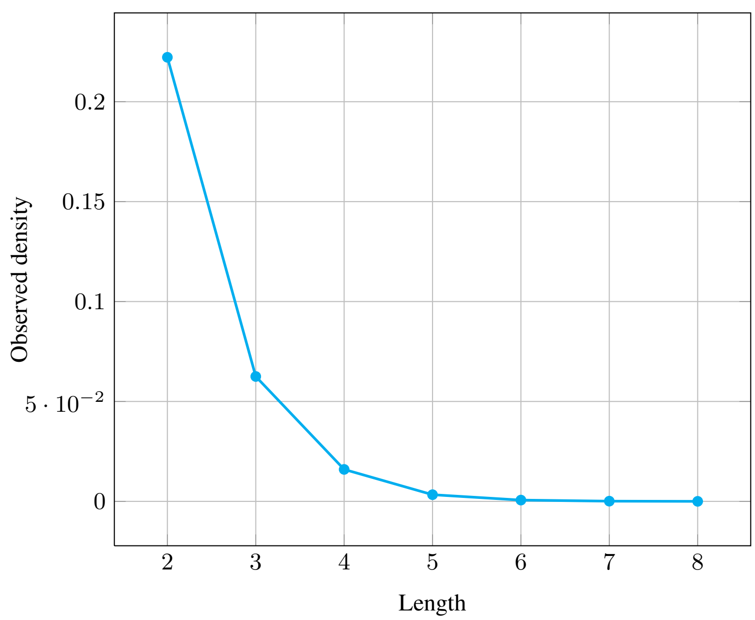
Length (n)	2	3	4	5	6	7	8
Solutions	2	4	10	26	76	232	764

Number of solutions for `symmetric_alldifferent_except_0`: domains $0..n$

Solution density for `symmetric_alldifferent_except_0`



Solution density for `symmetric_alldifferent_except_0`



See also **implied by:** `symmetric_alldifferent`.

implies (items to collection): `k_alldifferent`, `lex_alldifferent`.

Keywords **application area:** sport timetabling.

characteristic of a constraint: joker value.

combinatorial object: matching.

constraint type: predefined constraint, timetabling constraint.

Cond. implications symmetric_alldifferent_except_0(NODES)
implies alldifferent_except_0(VARIABLES : NODES).