UCA - Master Hydroprotech - 2026

Mathematical optimization: applications to water management

Sophie Demassey
https://sofdem.github.io/

MINES PARIS | PSL
Centre de Mathématiques Appliquées

study cases

groundwater abstraction

urban water networks

hydroelectricity production

Groundwater

credit: nasa.gov

ex: sustainable abstraction

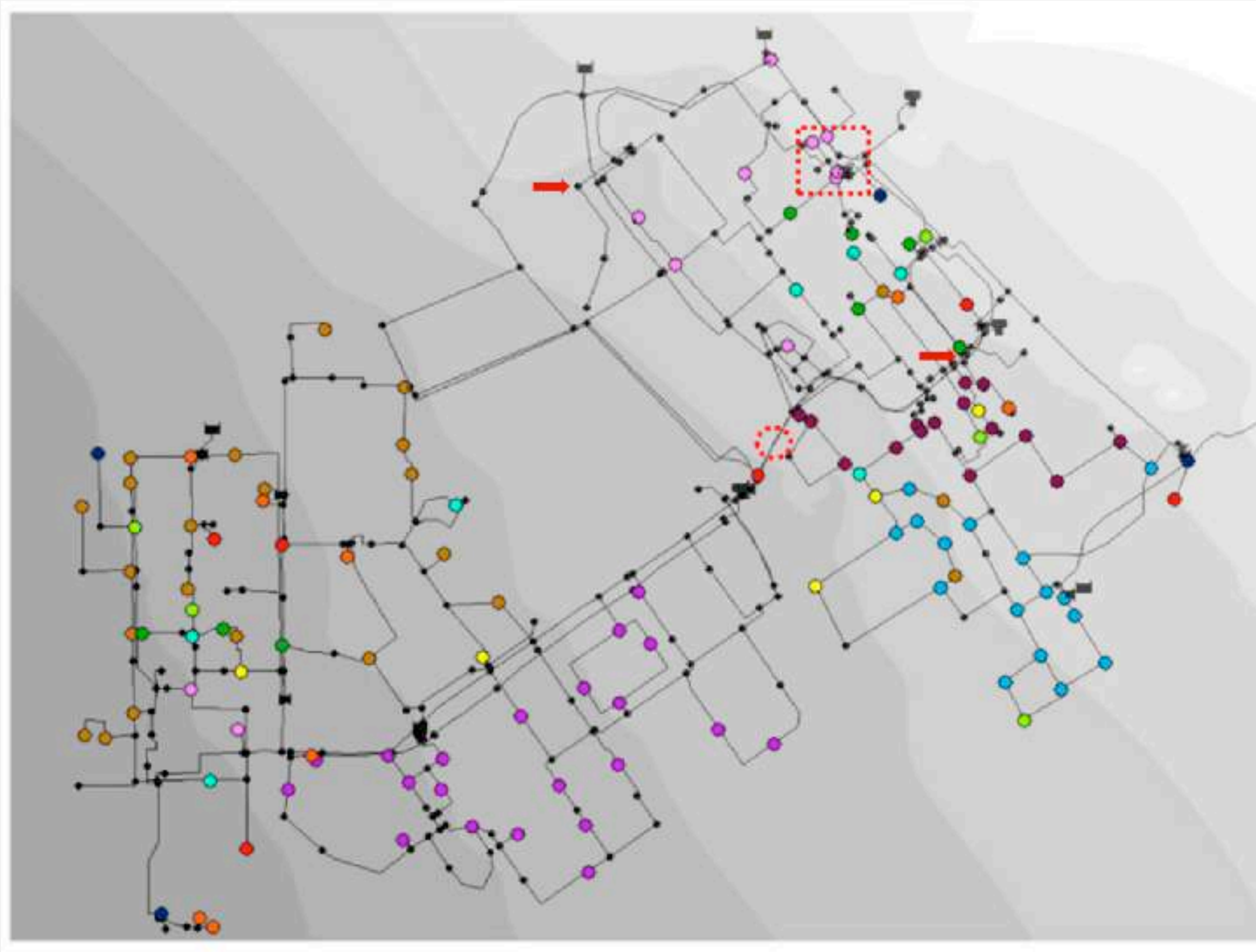
place pumps and plan pumping
to prevent aquifer depletion (then land subsidence or seawater intrusion)
and quality degradation (temperature, salinity)
while maximizing the abstraction value

strong uncertainties (aquifer recharge rate), approximate dynamics(quality) and sustainability models

[Hassan et al. Mapping the optimization of groundwater abstraction research: A bibliometric review in the context of South Asian region. Heliyon 2023]

Urban networks

ex: pipe sizing



select the size of the pipes in a gravity-fed network
to satisfy the demand at each delivery node
while minimizing the installation costs

finite catalog of pipes: size ↗ capacity ↗ cost ↗

ex: pipe sizing

$x_{ak} \in \{0,1\}$ assign size k to pipe a

q_a flow in pipe a, $v_a = h_i - h_j$ head loss in pipe a, quadratic resistance $v_a = \phi_a(q_a)$

$(q,h) = NAP(D,H,\phi)$ hydraulic equilibrium w.r.t demand D, head H, resistance ϕ

$$\min_{x,q,h} \sum_a \sum_k c_{ak} x_{ak}$$

$$s.t. x_{ak} = 0 \implies q_{ak} = v_{ak} = 0 \qquad \forall a \in A, k \in K$$

$$\sum_k x_{ak} = 1, h_i - h_j = \sum_k v_{ak} \qquad \forall a = (i,j) \in A$$

$$(q_{AK}, h_S) \in NAP(D_S, H_R, \phi_{AK(x)}).$$

bilevel program or
simulation-based genetic algorithm

nonconvex MINLP or approximate MILP

ex: pipe sizing

convex MINLP reformulation

$$\min_{x,q,h} \sum_a \sum_k c_{ak} x_{ak}$$

$$\text{s.t.} x_{ak} = 0 \implies q_{ak} = v_{ak} = 0 \qquad \forall a \in A, k \in K$$

$$\sum_k x_{ak} = 1, h_i - h_j = \sum_k v_{ak} \qquad \forall a = (i,j) \in A$$

$$\sum_{ak} E_{as} q_{ak} = D_s \qquad \forall s \in S$$

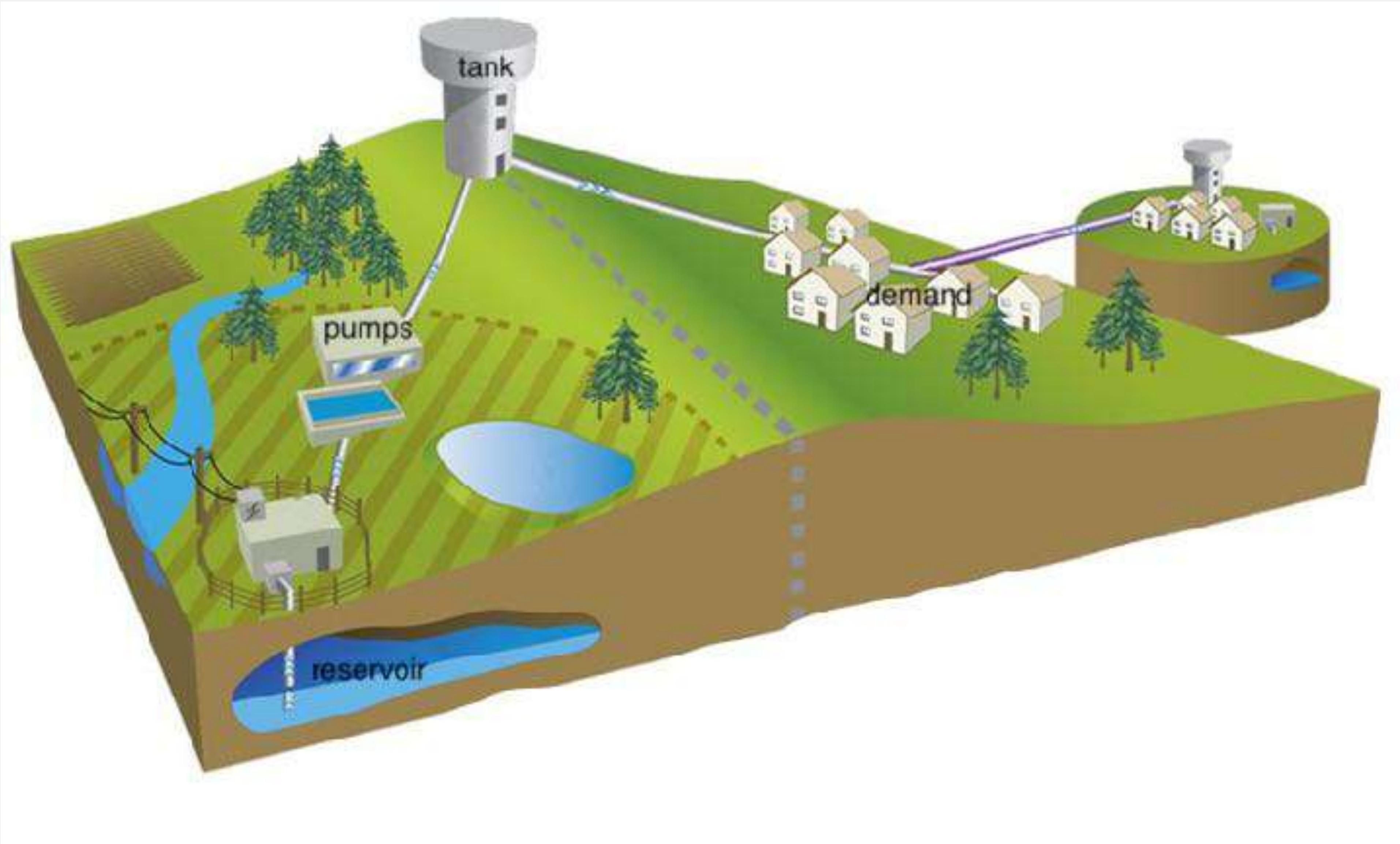
$$\sum_{ak} \left(f_{ak}(q_{ak}) + f^*_{ak}(v_{ak}) \right) + H_R^\top q_R + D_S^\top h_S \leq 0 \qquad (SD)$$

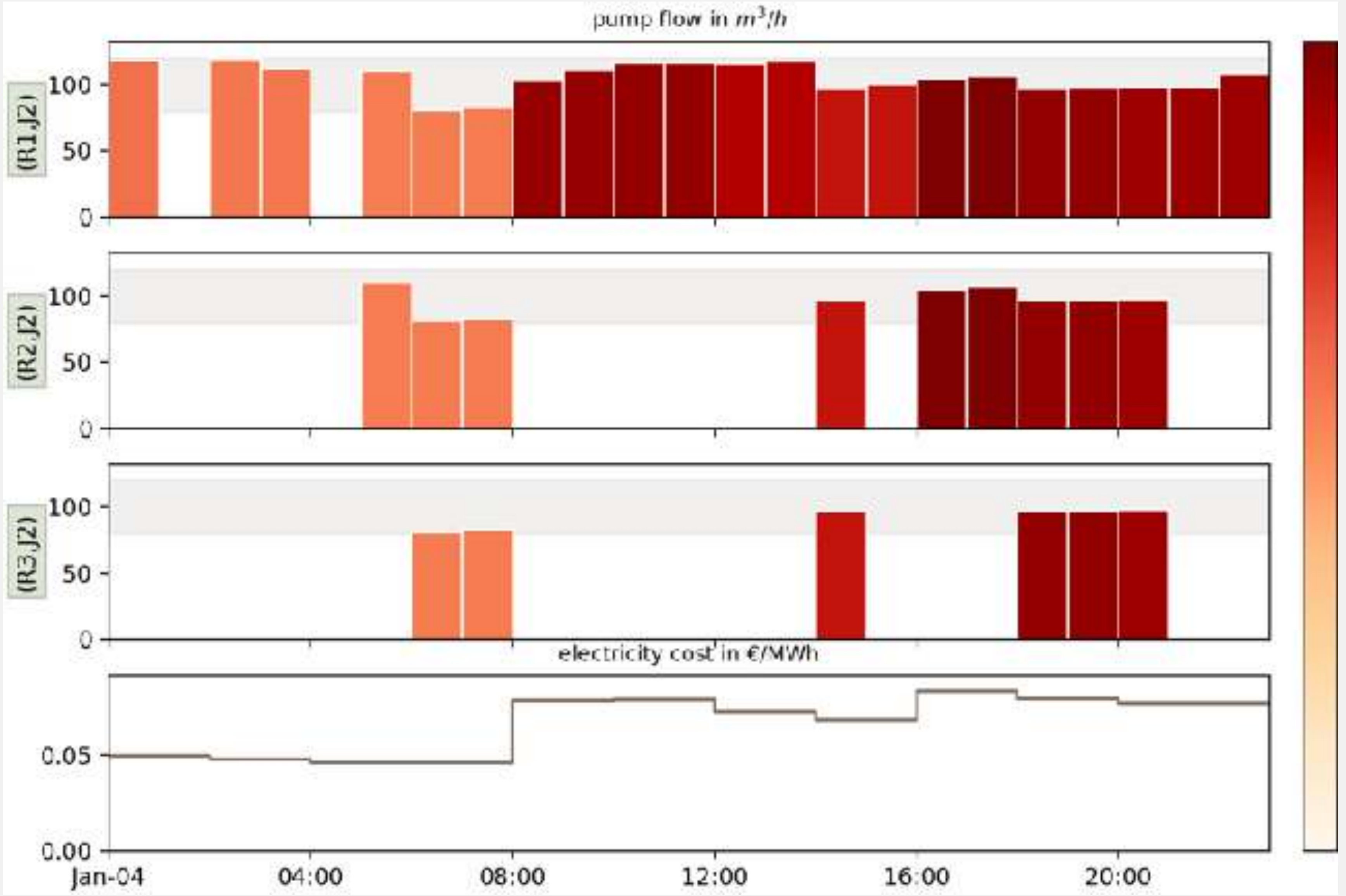
$f = \int \phi, f^* = \int \phi^{-1}$ are convex but not quadratic

[Demassey. Strong duality reformulation for bilevel optimization over nonlinear flow networks. 2023]

ex: pump scheduling
(load shifting in pressurized networks)

schedule pumps and valves in a pressurized network on a time horizon
to satisfy the varying demand at each delivery node
and the capacity of the water tanks
while minimizing the electricity bill





ex2: pump scheduling

$x_{at} \in \{0,1\}$ activate pump/valve a at time t

hydraulic equilibrium in the active subnetwork

limit the water tank level H

$$\min \sum_a \sum_t c^0_{at} x_{at} + c^1_{at} q_{at}$$

$$\begin{aligned}
&s.t. (q_{At}, h_{St}) \in NAP(D_{St}, H_{Rt}, \phi_{A(x_t)}) && \forall t \in T \\
&x_{at} = 0 \implies q_{at} = 0 && \forall a \in A, t \in T \\
&H_{R(t+1)} = H_{Rt} + s_R^\top q_{Rt} && \forall t \in T \\
&\underline{H}_{Rt} \leq H_{Rt} \leq \overline{H}_{Rt} && \forall t \in T.
\end{aligned}$$

additional complexity: temporal inter-dependency

[Demassey Strong duality reformulation for bilevel optimization over nonlinear flow networks. 2023]

water network optimization
(drinking, waste, irrigation)

decisions

dimension
renovation
extension
sectorization
scheduling operations
scheduling maintenance
place equipments and controllers
calibrate hydraulic models

concerns

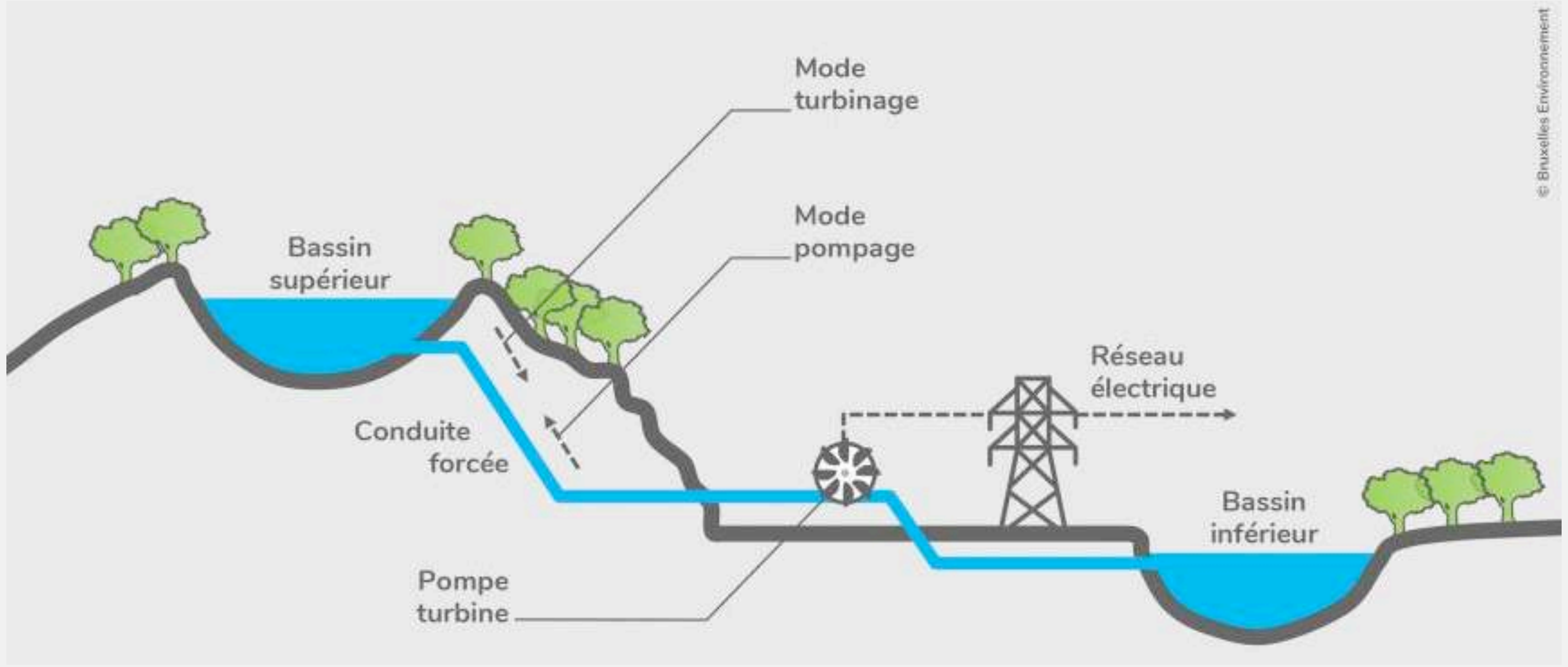
demand: standard, worst-case, emergency
network topology
energy consumption
leakage, over-pressure
flow conservation
pressure-flow relation
chlorine consumption
water quality, treatment
storage capacity
resilience to failures or storms
sewer overflow

[Bello, et al. Solving Management Problems in Water Distribution Networks: A Survey of Approaches and Mathematical Models. Water 2019]
[Mala-Jetmarova, Sultanova, Savic. Lost in Optimisation of Water Distribution Systems? A Literature Review of System Design. Water 2018]


Hydropower

ex: hydro unit commitment

schedule pumps and turbine
to ensure flow conservation
and maintain reservoir level in their limits
w.r.t strategic constraints (load balance, ramp, irrigation)
while maximizing the power production value



(lagrangian) subproblem of day-to-day unit commitment encompassing national power systems

ex4: hydro unit commitment

flow q_{it}, volume v_{it}, power production/consumption p_{it} in plant i at time t

nonlinear flow-power relation ϕ (turbine), disjunctive flow domains

volume conservation and limits in reservoirs

$$\max \sum_{i \in I} \sum_{t \in T} \lambda_{it} p_{it} \qquad (1)$$

$$p_{it} = \Phi(q_{it}, v_{it}) \qquad \forall t, \forall i \qquad (2)$$

$$v_{it} = v_{i(t-1)} + I_{it} + \Delta T(-q_{it} + \sum_{r \in I_i^+} q_{r(t-1)} - \sum_{r \in I_i^-} q_{r(t-1)}) \qquad \forall t, \forall i \qquad (3)$$

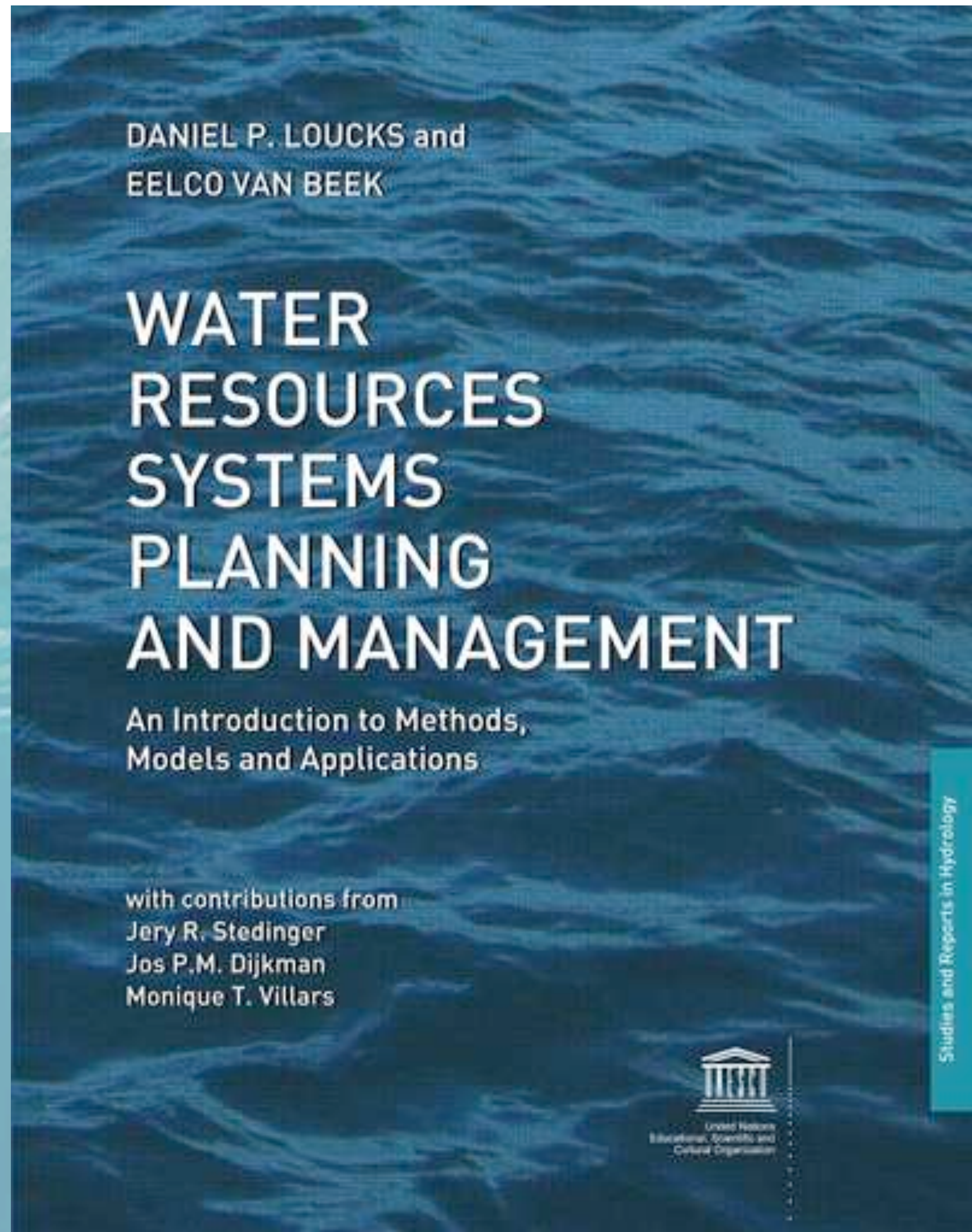
$$q_{it} \in \{Q_i^-\} \cup \{0\} \cup [\underline{Q}_i, \overline{Q}_i] \qquad \forall t, \forall i \qquad (4)$$

$$\underline{V}_i \leq v_{it} \leq \overline{V}_i \qquad \forall t, \forall i \qquad (5)$$

[Taktak & d'Ambrosio. An overview on mathematical programming approaches for the deterministic unit commitment problem in hydro valleys. Energy Sys 2017]

conclusion

- diversity of water systems & processes
- management involves decision
- from simulation (*what if*) to optimization (*what should*)
- difficulties: nonlinear dynamics, uncertain forecasts, intricated systems, fuzzy objectives
- trade-off between accurate models and efficient algorithms



Loucks & Beek (2005)
https://unesdoc.unesco.org/ark:/48223/pf0000143430

Mathematical optimization: introduction and application to water management

Sophie Demassey
https://sofdem.github.io/