water optimization

water is

a commodity, a resource, an environment

drinking water
wastewater
rain, ice, surface water, ground water
fresh, brackish, saline water
irrigation water
source of hydropower (river, tide, wave)
vector of pumped-storage hydroelectricity
steam to generate heat and energy
water for cooling or cleaning
water for processing (fracking, diluting, drilling)
storms, floods, droughts, mudflows, tsunamis
subject to thermal, chemical pollution
related to climate change, climate variability
wetlands, rain forests, oceans, coasts and rivers

to process

extract,
supply,
treat,
produce,
irrigate,
desalinate,
purify,
drain,
heat,
blend,
store,
pump,
flow,
preserve,
measure,
prevent,
control

in small/large systems

urban networks
sewers
desalination plants
farms
power systems
hydropower plants
thermal plants
industries
municipalities
pumps, turbines
aquifers
drainage basins
ecosystems
world

water optimization

Operational

organize the process

select elements to operate
assign operation level
allocate resources
schedule operations
position elements

Tactical

design the system

select elements to dimension, maintain
assign dimension, equipment
plan resources and times

often discrete decisions
nonlinear physical dynamics
minimize an economic/social/ecological cost

study cases

urban water networks

groundwater abstraction

hydroelectricity production

sustainable abstraction

place pumps and plan pumping
to prevent aquifer depletion (then land subsidence or seawater intrusion)
and quality degradation (temperature, salinity)
while maximizing the abstraction value

strong uncertainties (aquifer recharge rate), approximate dynamics(quality) and sustainability models

[Hassan et al. Mapping the optimization of groundwater abstraction research: A bibliometric review in the context of South Asian region. Heliyon 2023]

my first MILP

Groundwater abstraction (identical pumps)

Choose places to install pumps within a finite set J of candidates
and minimize the global cost, given:

- installation cost c_j and average flow rate q_j of a pump at place $j \in J$
- limited total abstraction rate: lower $\underline{Q}$ and upper $\overline{Q}$ limits
- at most 3 pumps installed, no 2 pumps on places a and $b \in J$

variables: $x_j \in \{0,1\}$ number of pumps installed at $j \in J$

$$
\begin{aligned}
& min \sum c_j x_j : \\
& \underline{Q} \leq \sum q_j x_j \leq \overline{Q}, \sum x_j \leq 3 \\
& x_a + x_b \leq 1,\ x \in \{0,1\}^J
\end{aligned}
$$

variant MILP

Groundwater abstraction (distinct available pumps)

Assign available pumps taken from a finite set K:

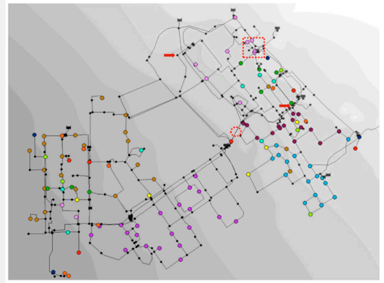
- installation cost c_k and flow rate q_k now depends on pump $k \in K$
- limited abstraction $\underline{Q}, \overline{Q}$

$$
\begin{aligned}
& min \sum_k \sum_j c_k x_{jk} : \\
& \underline{Q} \leq \sum_k \sum_j q_k x_{jk} \leq \overline{Q} \\
& \sum_j x_{jk} \leq 1 \quad \forall k \in K \\
& \sum_k x_{jk} \leq 1 \quad \forall j \in J \\
& x_{jk} \in \{0,1\} \quad \forall j \in J, k \in K
\end{aligned}
$$

variables: $x_{jk} = 1$ if pump $k \in K$ installed at place $j \in J$

pipe sizing

select the size of the pipes in a gravity-fed network
to satisfy the demand at each delivery node
while minimizing the installation costs



finite catalog of pipes: size ↗ capacity ↗ cost ↗

pipe sizing

assign a size k to each pipe a: $x_{ak} = 1$ (otherwise $x_{ak} = 0$)
hydraulic equilibrium between flows q and heads h, v in the selected network

$$
\begin{aligned}
&\min_{x,q,h} \sum_a \sum_k c_{ak} x_{ak} \\
&\text{s.t. } x_{ak} = 0 \implies q_{ak} = v_{ak} = 0 && \forall a \in A, k \in K \\
&\sum_k x_{ak} = 1, h_i - h_j = \sum_k v_{ak} && \forall a = (i,j) \in A \\
&(q_{AK}, h_S) \in NAP(D_S, H_R, \phi_{AK(x)}).
\end{aligned}
$$

bilevel program or
simulation-based genetic algorithm

convex MINLP or approximate MILP
+ branch-and-bound

pipe sizing

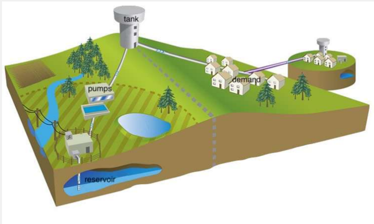
convex MINLP reformulation

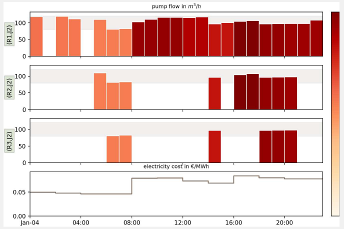
$$
\begin{aligned}
&\min_{x,q,h} \sum_a \sum_k c_{ak} x_{ak} \\
&\text{s.t. } x_{ak} = 0 \implies q_{ak} = v_{ak} = 0 && \forall a \in A, k \in K \\
&\sum_k x_{ak} = 1, h_i - h_j = \sum_k v_{ak} && \forall a = (i,j) \in A \\
&\sum_{ak} E_{as} q_{ak} = D_s && \forall s \in S \\
&\sum_{ak} \left(f_{ak}(q_{ak}) + f^*_{ak}(v_{ak}) \right) + H_R^\top q_R + D_S^\top h_S \leq 0 && (SD)
\end{aligned}
$$

[Demassey. Strong duality reformulation for bilevel optimization over nonlinear flow networks. 2023]

pump scheduling
(load shifting in pressurized networks)

schedule pumps and valves in a pressurized network on a time horizon
to satisfy the varying demand at each delivery node
and the capacity of the water tanks
while minimizing the electricity bill





pump scheduling

activate pump/valve a at time t: $x_{at} = 1$ (otherwise $x_{at} = 0$)
hydraulic equilibrium between flows q and heads h, v in the active network
limit the water tank level H

$$\min \sum_a \sum_t c^0_{at} x_{at} + c^1_{at} q_{at}$$

$$\text{s.t.} (q_{At}, h_{St}) \in NAP(D_{St}, H_{Rt}, \phi_{A(x_t)}) \quad \forall t \in T$$

$$x_{at} = 0 \implies q_{at} = 0 \quad \forall a \in A, t \in T$$

$$H_{R(t+1)} = H_{Rt} + s^{\top}_R q_{Rt} \quad \forall t \in T$$

$$\underline{H}_{Rt} \leq H_{Rt} \leq \overline{H}_{Rt} \quad \forall t \in T.$$

additional complexity: temporal inter-dependency

[Demassey Strong duality reformulation for bilevel optimization over nonlinear flow networks. 2023]

water network optimization
(drinking, waste, irrigation)

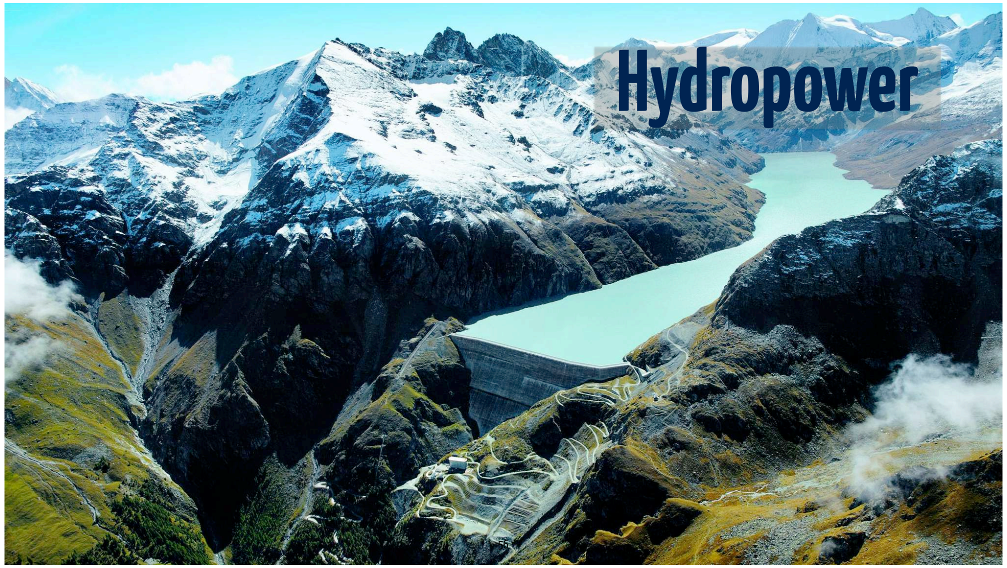
decisions

- dimension
- renovation
- extension
- sectorization
- scheduling operations
- scheduling maintenance
- place equipments and controllers
- calibrate hydraulic models

concerns

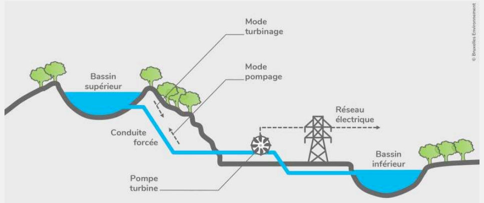
- demand: standard, worst-case, emergency
- network topology
- energy consumption
- leakage, over-pressure
- flow conservation
- pressure-flow relation
- chlorine consumption
- water quality, treatment
- storage capacity
- resilience to failures or storms
- sewer overflow

[Bello, et al. Solving Management Problems in Water Distribution Networks: A Survey of Approaches and Mathematical Models. Water 2019]
[Mala-Jetmarova, Sultanova, Savic. Lost in Optimisation of Water Distribution Systems? A Literature Review of System Design. Water 2018]



hydro unit commitment

schedule pumps and turbine
to ensure flow conservation
and maintain reservoir level in their limits
w.r.t strategic constraints (load balance, ramp, irrigation)
while maximizing the power production value



(lagrangian) subproblem of day-to-day unit commitment encompassing national power systems

hydro unit commitment

flow q_{it}, volume v_{it}, power production/consumption p_{it} in plant i at time t
nonlinear flow-power relation ϕ (turbine), disjunctive flow domains
volume conservation and limits in reservoirs

$$\max \sum_{i\in I}\sum_{t\in T} \lambda_{it} p_{it} \qquad (1)$$

$$p_{it} = \Phi(q_{it}, v_{it}) \quad \forall t, \forall i \qquad (2)$$

$$v_{it} = v_{i(t-1)} + I_{it} + \Delta T(-q_{it} + \sum_{r\in I_i^+} q_{r(t-1)} - \sum_{r\in I_i^-} q_{r(t-1)}) \quad \forall t, \forall i \qquad (3)$$

$$q_{it} \in \{Q_i^-\} \cup \{0\} \cup [\underline{Q}_i, \overline{Q}_i] \quad \forall t, \forall i \qquad (4)$$

$$\underline{V}_i \le v_{it} \le \overline{V}_i \quad \forall t, \forall i \qquad (5)$$

[Taktak & d'Ambrosio. An overview on mathematical programming approaches for the deterministic unit commitment problem in hydro valleys. Energy Sys 2017]

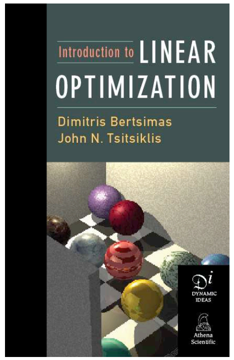
conclusion

- huge diversity of water systems & processes
- management involves decision involves optimization, e.g. maximize sustainability
- mathematical optimization as a low-tech solution (except computation and data acquisition) to get as much out of existing investments
- uncertain forecasts, intricated systems, nonlinear dynamics, fuzzy objectives: trade-off between accurate models and efficient algorithms

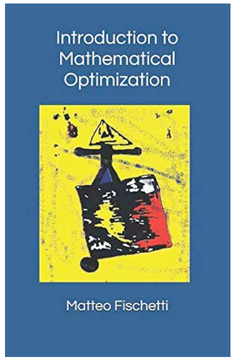
future

- modelling sustainability accurately
- from simulation (*what if*) to optimization (*what should*)
- short/long-term model coupling: time-scale reconciliation

D. Bertsimas & J. Tsitsiklis 1997



M. Fischetti 2019



L. Wolsey 1998